
JOHN BIRD
SCIENCE AND MATHEMATICS FOR ENGINEERING
Sixth Edition
ROUTLEDGE

Science and Mathematics for Engineering

Why is knowledge of science and mathematics important in engineering?

A career in any engineering field will require both basic and advanced mathematics and science. Without mathematics and science to determine principles, calculate dimensions and limits, explore variations, prove concepts, and so on, there would be no mobile telephones, televisions, stereo systems, video games, microwave ovens, computers, or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical. There would be no metals beyond the common ones, such as iron and copper, no plastics, no synthetics. In fact, society would most certainly be less advanced without the use of mathematics and science throughout the centuries and into the future.

Electrical engineers require mathematics and science to design, develop, test, or supervise the manufacturing and installation of electrical equipment, components, or systems for commercial, industrial, military, or scientific use.

Mechanical engineers require mathematics and science to perform engineering duties in planning and designing tools, engines, machines, and other mechanically functioning equipment; they oversee installation, operation, maintenance, and repair of such equipment as centralised heat, gas, water, and steam systems.

Aerospace engineers require mathematics and science to perform a variety of engineering work in designing, constructing, and testing aircraft, missiles, and spacecraft; they conduct basic and applied research to evaluate adaptability of materials and equipment to aircraft design and manufacture and recommend improvements in testing equipment and techniques.

Nuclear engineers require mathematics and science to conduct research on nuclear engineering problems or apply principles and theory of nuclear science to problems concerned with release, control, and utilisation of nuclear energy and nuclear waste disposal.

Petroleum engineers require mathematics and science to devise methods to improve oil and gas well production and determine the need for new or modified tool designs; they oversee drilling and offer technical advice to achieve economical and satisfactory progress.

Industrial engineers require mathematics and science to design, develop, test, and evaluate integrated systems for managing industrial production processes, including human work factors, quality control, inventory control, logistics and material flow, cost analysis, and production coordination.

Environmental engineers require mathematics and science to design, plan, or perform engineering duties in the prevention, control, and remediation of environmental health hazards, using various engineering disciplines; their work may include waste treatment, site remediation, or pollution control technology.

Civil engineers require mathematics and science in all levels in civil engineering – structural engineering, hydraulics and geotechnical engineering are all fields that employ mathematical tools such as differential equations, tensor analysis, field theory, numerical methods and operations research.

Knowledge of mathematics and science is therefore needed by each of the engineering disciplines listed above.

It is intended that this text – *Science and Mathematics for Engineering* – will provide a step by step approach to learning fundamental mathematics and science needed for your engineering studies.

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Sixth Edition

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To Sue

Contents

Preface

'Science and Mathematics for Engineering, 6th Edition', aims to develop in the reader an understanding of fundamental scientific and applied mathematical principles which will enable the solution of elementary engineering problems. The aims are to describe engineering systems in terms of basic scientific laws and principles, to investigate the behaviour of simple linear systems in engineering, to calculate the response of engineering systems to changes in variables, and to determine the response of such engineering systems to changes in parameters. In particular, the aim is to develop an understanding of applied mathematics, static's, dynamics, electrical principles, energy and engineering systems.

In this sixth edition of *Science and Mathematics for Engineering*, (formerly entitled *Science for Engineering*), additional material on metric to imperial conversions, logarithms and exponential functions, heat exchangers, resistor colour coding, lithium-ion and glass batteries, solar energy, rectification, global climate change and the generation of electricity have all been added to the text.

The website **http://www.routledge.com/cw/bird** contains full solutions to all of the problems in the text, together with much more – see page xiv.

The text covers the following courses of study:

(i) **Mathematics for Engineering Technicians** (BTEC First Certificate/Diploma, level 2)

(ii) **Applied Electrical and Mechanical Science for Engineering** (BTEC National Certificate/Diploma, level 2)

(iii) **Mathematics for Engineering Technicians** (BTEC National Certificate/Diploma, level 3)

(iv) **Any introductory/access/foundation course** involving Engineering Science and basic Mathematics

This sixth edition of *Science and Mathematics for Engineering* is arranged in four sections.

Section 1, Applied Mathematics, chapters 1 to 14, provides the basic mathematical tools needed to effectively understand the science applications in sections II, III and IV. Basic arithmetic, fractions, decimals, percentages, indices, units, prefixes and engineering notation, calculations and evaluation of formulae, algebra, simple equations, transposition of formulae, simultaneous equations, logarithms and exponential functions, straight line graphs, trigonometry, areas of common shapes, the circle, and volumes of common solids are covered in this first section.

Section II, Mechanical Applications, chapters, 15 to 33, covers SI units, density, atomic structure of matter, speed and velocity, acceleration, forces acting at a point, work, energy and power, simply supported beams, linear and angular motion, friction, simple machines, the effects of forces on materials, linear momentum and impulse, torque, pressure in fluids, heat energy and transfer, thermal expansion, ideal gas laws and the measurement of temperature.

Section III, Electrical Applications, chapters 34 to 44, covers an introduction to electric circuits, resistance variation, batteries and alternative sources of energy, series and parallel networks, Kirchhoff's laws, magnetism and electromagnetism, electromagnetic induction, alternating voltages and currents, capacitors and inductors, electrical measuring instruments and measurements, and global climate change and future electricity production.

Section IV, Engineering systems, chapter 45, covers an overview of the principles of electronic and mechanical engineering systems, forming a basis for further studies.

Each topic in the text is presented in a way that assumes in the reader little previous knowledge of that topic. Theory is introduced in each chapter by an outline of essential information, definitions, formulae, laws and procedures. The theory is kept to a minimum, for problem solving is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

Science and Mathematics for Engineering, 6th Edition contains some **650 worked problems**, together with **430 multiple-choice questions,** and over **1440 further**

questions, arranged in **219 Exercises**, all with answers at the back of the book; the Exercises appear at regular intervals – every 2 or 3 pages – throughout the text. Also included are **432 short answer questions**, the answers for which can be determined from the preceding material in that particular chapter**. 463 line diagrams** further enhance the understanding of the theory. All of the problems – multiple-choice, short answer and further questions – mirror where possible practical situations in science and engineering.

Full of solutions to the further problems and a **PowerPoint presentation of all the illustrations** contained in the text is available on the website – see below.

At regular intervals throughout the text are fifteen **Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in chapters 1 and 2, Revision Test 2 covers the material contained in chapters 3 and 4, and so on. These Revision Tests do not have answers given since it is envisaged that lecturers/instructors could set the tests for students to attempt as part of their course structure. Lecturers' may obtain solutions of the Revision Tests on the website – see below.

A list of the **main formulae** is included at the end of the book for easy reference, together with a comprehensive **glossary of terms**.

'Learning by Example' is at the heart of '**Science and Mathematics for Engineering 6th Edition**'.

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The following support material is available from **http://www.routledge.com/cw/bird**

For Students:

1. **Full worked solutions to over 1440 further questions contained in the 219 Practice Exercises**
2. **A list of Essential Formulae**
3. **A full glossary of terms**
4. **430 multiple-choice questions**
5. **Information on 38 Famous Engineers and Scientists mentioned in the text**

For Lecturers/Instructors:

1–5. As per students 1–5 above

6. **Full solutions and marking scheme for each of the 15 Revision Tests; also, each test may be downloaded for distribution to students.**
7. **All 463 illustrations used in the text may be downloaded for use in PowerPoint presentations**

Section I

Applied mathematics

Chapter 1

Basic arithmetic

Why it is important to understand: **Basic arithmetic**

Being numerate, i.e. having an ability to add, subtract, multiply and divide whole numbers with some confidence, goes a long way towards helping you become competent at mathematics. Of course electronic calculators are a marvellous aid to the quite complicated calculations often required in engineering; however, having a feel for numbers ìn our head' can be invaluable when estimating. Do not spend too much time on this chapter because we deal with the calculator later; however, try to have some idea how to do quick calculations in the absence of a calculator. You will feel more confident in dealing with numbers and calculations if you can do this.

At the end of this chapter, you should be able to:

- understand positive and negative integers
- add and subtract whole numbers
- multiply and divide two whole numbers
- multiply numbers up to 12×12 by rote
- determine the highest common factor from a set of numbers
- determine the lowest common factor from a set of numbers
- appreciate the order of operation when evaluating expressions
- understand the use of brackets in expressions
- evaluate expressions containing $+$, $-$, $\times$, $\div$ and brackets

1.1 Introduction

Whole Numbers

Whole Numbers are simply the numbers **0**, **1**, **2**, **3**, **4**, **5**, . . .

Counting Numbers

Counting Numbers are whole numbers, but **without the zero**, i.e. **1**, **2**, **3**, **4**, **5**, . . .

Natural Numbers

Natural Numbers can mean either counting numbers or whole numbers.

Integers

Integers are like whole numbers, but they **also include negative numbers**
Examples of integers include . . . − 5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, . . .

Arithmetic Operators

The four basic arithmetic operators are: add (+), subtract (−), multiply (×) and divide (÷).
It is assumed that adding, subtracting, multiplying and dividing reasonably small numbers can be achieved without a calculator. However, if revision of this area is needed then some worked problems are included in the following sections.
When **unlike signs** occur together in a calculation, the overall sign is **negative**.
For example,

$$3 + (-4) = 3 + -4 = 3 - 4 = \mathbf{-1}$$

and

$$(+5) \times (-2) = \mathbf{-10}$$

Like signs together give an overall **positive sign**. For example,

$$3 - (-4) = 3 - -4 = 3 + 4 = \mathbf{7}$$

and

$$(-6) \times (-4) = \mathbf{+24}$$

Prime Numbers

A prime number can be divided, without a remainder, only by itself and by 1. For example, 17 can be divided only by 17 and by 1. Other examples of prime numbers are 2, 3, 5, 7, 11, 13, 19 and 23.

1.2 Revision of addition and subtraction

You can probably already add two or more numbers together and subtract one number from another. However, if you need revision of this, then the following worked problems should be helpful.

Problem 1. Determine 735 + 167

	H	T	U
	7	3	5
+	1	6	7
	9	0	2
	1	1	

(i) $5 + 7 = 12$. Place the 2 in the units (U) column. Carry the 1 in the tens (T) column.

(ii) $3 + 6 + 1$ (carried) $= 10$. Place the 0 in the tens column. Carry the 1 in the hundreds (H) column.

(iii) $7 + 1 + 1$ (carried) $= 9$. Place the 9 in the hundreds column.

Hence, $\mathbf{735 + 167 = 902}$

Problem 2. Determine 632 − 369

	H	T	U
	6	3	2
−	3	6	9
	2	6	3

(i) $2 - 9$ is not possible; therefore 'borrow' 1 from the tens column (leaving 2 in the tens column). In the units column, this gives us $12 - 9 = 3$

(ii) Place 3 in the units column.

(iii) $2 - 6$ is not possible; therefore 'borrow' 1 from the hundreds column (leaving 5 in the hundreds column). In the tens column, this gives us $12 - 6 = 6$

(iv) Place the 6 in the tens column.

(v) $5 - 3 = 2$

(vi) Place the 2 in the hundreds column.

Hence, $\mathbf{632 - 369 = 263}$

Problem 3. Add 27, −74, 81 and −19

This problem is written as $27 - 74 + 81 - 19$.

Adding the positive integers:	27
	81
Sum of positive integers is:	108
Adding the negative integers:	74
	19
Sum of negative integers is:	93
Taking the sum of the negative integers from the sum of the positive integers gives:	108
	−93
	15

Thus, $\mathbf{27 - 74 + 81 - 19 = 15}$

Problem 4. Subtract −74 from 377

This problem is written as 377 − −74. Like signs together give an overall positive sign, hence

$$377 - -74 = 377 + 74$$

$$\begin{array}{r} 377 \\ +\ 74 \\ \hline 451 \\ \hline \end{array}$$

Thus, **377 − −74 = 451**

Now try the following Practice Exercise

Practice Exercise 1 Further problems on addition and subtraction (answers on page 531)

In problems 1–11, determine the values of the expressions given, without using a calculator:

1. 67 kg − 82 kg + 34 kg
2. 851 mm − 372 mm
3. 124 − 273 + 481 − 398
4. £927 − £114 + £182 − £183 − £247
5. 647 − 872
6. 2417 − 487 + 2424 − 1778 − 4712
7. £2715 − £18,250 + £11,471 − £1509 + £113,274
8. 47 + (−74) − (−23)
9. 813 − (−674)
10. −23,148 − 47,724
11. $53,774 − $38,441

1.3 Revision of multiplication and division

You can probably already multiply two numbers together and divide one number by another. However, if you need to revise this then the following worked problems should be helpful.

Problem 5. Determine 86 × 7

$$\begin{array}{r} \mathbf{H\ T\ U} \\ 8\ 6 \\ \times\quad 7 \\ \hline 6\ 0\ 2 \\ \hline 4 \end{array}$$

(i) 7 × 6 = 42. Place the 2 in the units (U) column and 'carry' the 4 into the tens (T) column.

(ii) 7 × 8 = 56; 56 + 4 (carried) = 60. Place the 0 in the tens column and the 6 in the hundreds (H) column.

Hence, **86 × 7 = 602**

A good grasp of **multiplication tables** is needed when multiplying such numbers; a reminder of the multiplication table up to 12 × 12 is shown on page 6. Confidence in handling numbers will be greatly improved if this table is memorised.

Problem 6. Determine 764 × 38

$$\begin{array}{r} 764 \\ \times\ 38 \\ \hline 6112 \\ 22920 \\ \hline 29032 \\ \hline \end{array}$$

(i) 8 × 4 = 32. Place the 2 in the units column and carry 3 into the tens column.

(ii) 8 × 6 = 48; 48 + 3 (carried) = 51. Place the 1 in the tens column and carry the 5 into the hundreds column.

(iii) 8 × 7 = 56; 56 + 5 (carried) = 61. Place the 1 in the hundreds column and the 6 in the thousands column.

(iv) Place 0 in the units column under the 2

(v) 3 × 4 = 12. Place the 2 in the tens column and carry the 1 into the hundreds column.

(vi) 3 × 6 = 18; 18 + 1 (carried) = 19. Place the 9 in the hundreds column and carry the 1 into the thousands column.

(vii) 3 × 7 = 21; 21 + 1 (carried) = 22. Place a 2 in the thousands column and the other 2 in the ten thousands column.

(viii) 6112 + 22,920 = 29,032

Multiplication table

×	2	3	4	5	6	7	8	9	10	11	12
2	**4**	6	8	10	12	14	16	18	20	22	24
3	6	**9**	12	15	18	21	24	27	30	33	36
4	8	12	**16**	20	24	28	32	36	40	44	48
5	10	15	20	**25**	30	35	40	45	50	55	60
6	12	18	24	30	**36**	42	48	54	60	66	72
7	14	21	28	35	42	**49**	56	63	70	77	84
8	16	24	32	40	48	56	**64**	72	80	88	96
9	18	27	36	45	54	63	72	**81**	90	99	108
10	20	30	40	50	60	70	80	90	**100**	110	120
11	22	33	44	55	66	77	88	99	110	**121**	132
12	24	36	48	60	72	84	96	108	120	132	**144**

Hence, **764 × 38 = 29,032**

Again, knowing multiplication tables is rather important when multiplying such numbers.

It is appreciated, of course, that such a multiplication can, and probably will, be performed using a **calculator**. However, there are times when a calculator may not be available and it is then useful to be able to calculate the 'long way'.

Problem 7. Determine 1834 ÷ 7

$$7\overline{)1834}\quad 262$$

(i) 7 into 18 goes 2, remainder 4. Place the 2 above the 8 of 1834 and carry the 4 remainder to the next digit on the right, making it 43.

(ii) 7 into 43 goes 6, remainder 1. Place the 6 above the 3 of 1834 and carry the 1 remainder to the next digit on the right, making it 14.

(iii) 7 into 14 goes 2, remainder 0. Place 2 above the 4 of 1834.

Hence, $\mathbf{1834 \div 7 = 1834/7 = \frac{1834}{7} = 262}$

The method shown is called **short division**.

Problem 8. Determine 5796 ÷ 12

$$\begin{array}{r} 483 \\ 12\overline{)5796} \\ \underline{48} \\ 99 \\ \underline{96} \\ 36 \\ \underline{36} \\ 00 \end{array}$$

(i) 12 into 5 won't go. 12 into 57 goes 4; place 4 above the 7 of 5796

(ii) 4 × 12 = 48; place the 48 below the 57 of 5796

(iii) 57 − 48 = 9

(iv) Bring down the 9 of 5796 to give 99

(v) 12 into 99 goes 8; place 8 above the 9 of 5796

(vi) 8 × 12 = 96; place 96 below the 99

(vii) 99 − 96 = 3

(viii) Bring down the 6 of 5796 to give 36

(ix) 12 into 36 goes 3 exactly.

(x) Place the 3 above the final 6

(xi) 3 × 12 = 36; place the 36 below the 36

(xii) 36 − 36 = 0

Hence, $\mathbf{5796 \div 12 = 5796/12 = \frac{5796}{12} = 483}$

The method shown is called **long division**.

Now try the following Practice Exercise

Practice Exercise 2 Further problems on multiplication and division (answers on page 531)

Determine the values of the expressions given in problems 1–7, without using a calculator:

1. (a) 78×6 (b) 124×7
2. (a) £261 × 7 (b) £462 × 9
3. (a) 783 kg × 11 (b) 73 kg × 8
4. (a) 27 mm × 13 (b) 77 mm × 12
5. (a) 288 m ÷ 6 (b) 979 m ÷ 11
6. (a) $\frac{1813}{7}$ (b) $\frac{896}{16}$
7. (a) $\frac{88737}{11}$ (b) $46858 \div 14$
8. A screw has a mass of 15 grams. Calculate, in kilograms, the mass of 1200 such screws (1 kg = 1000 g).
9. Holes are drilled 36 mm apart in a metal plate. If a row of 26 holes is drilled, determine the distance, in centimetres, between the centres of the first and last holes.
10. A builder needs to clear a site of bricks and top soil. The total weight to be removed is 696 tonnes. Trucks can carry a maximum load of 24 tonnes. Determine the number of truck loads needed to clear the site.

1.4 Highest common factors and lowest common multiples

When two or more numbers are multiplied together, the individual numbers are called **factors**. Thus a factor is a number which divides into another number exactly. The **highest common factor (HCF)** is the largest number which divides into two or more numbers exactly.

For example, consider the numbers 12 and 15.

The factors of 12 are 1, 2, 3, 4, 6 and 12 (i.e. all the numbers that divide into 12).

The factors of 15 are 1, 3, 5 and 15 (i.e. all the numbers that divide into 15).

1 and 3 are the only **common factors**, i.e. numbers which are factors of **both** 12 and 15.

Hence, **the HCF of 12 and 15 is 3** since 3 is the highest number which divides into **both** 12 and 15.

A **multiple** is a number which contains another number an exact number of times. The smallest number which is exactly divisible by each of two or more numbers is called the **lowest common multiple (LCM)**.

For example, the multiples of 12 are 12, 24, 36, 48, 60, 72, . . .

and the multiples of 15 are 15, 30, 45, 60, 75, . . .

60 is a common multiple (i.e. a multiple of **both** 12 and 15) and there are no lower common multiples.

Hence, **the LCM of 12 and 15 is 60** since 60 is the lowest number that both 12 and 15 divide into.

Here are some further problems involving the determination of HCFs and LCMs.

Problem 9. Determine the HCF of the numbers 12, 30 and 42

Probably the simplest way of determining an HCF is to express each number in terms of its lowest factors. This is achieved by repeatedly dividing by the prime numbers 2, 3, 5, 7, 11, 13, . . . (where possible) in turn. Thus

$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$

The factors which are common to each of the numbers are 2 in column 1 and 3 in column 3, shown by the dotted lines. Hence, the **HCF is 2 × 3**, i.e. **6**. That is, 6 is the largest number which will divide into 12, 30 and 42.

Problem 10. Determine the LCM of the numbers 12, 42 and 90

The LCM is obtained by finding the lowest factors of each of the numbers, as shown in Problem 9 above, and then selecting the largest group of any of the factors present. Thus

$12 = \boxed{2 \times 2} \times 3$

$42 = 2 \quad \times 3 \quad \times \boxed{7}$

$90 = 2 \quad \times \boxed{3 \times 3} \times \boxed{5}$

The largest group of any of the factors present are shown by the dotted lines and are 2×2 in 12, 3×3 in 90, 5 in 90 and 7 in 42.
Hence, **the LCM is $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$**, and is the smallest number which 12, 42 and 90 will all divide into exactly.

Now try the following Practice Exercise

Practice Exercise 3 Further problems on highest common factors and lowest common multiples (answers on page 531)

Find (a) the HCF and (b) the LCM of the following groups of numbers:

1. 8, 12
2. 60, 72
3. 50, 70
4. 270, 900
5. 6, 10, 14
6. 12, 30, 45
7. 10, 15, 70, 105
8. 90, 105, 300

1.5 Order of operation and brackets

1.5.1 Order of operation

Sometimes addition, subtraction, multiplication, division, powers and brackets may all be involved in a calculation. For example,

$$5 - 3 \times 4 + 24 \div (3 + 5) - 3^2$$

This is an extreme example but will demonstrate the order that is necessary when evaluating.
When we read, we read from left to right. However with mathematics there is a definite order of precedence which we need to adhere to.
The order is as follows:

Brackets
Order (or p**O**wer)
Division
Multiplication
Addition
Subtraction

Notice that the first letters of each word spell **BODMAS**, a handy *aide-mémoire*.
Order means p**O**wer. For example, $4^2 = 4 \times 4 = 16$

$$5 - 3 \times 4 + 24 \div (3 + 5) - 3^2$$

is evaluated as follows:

$5 - 3 \times 4 + 24 \div (3 + 5) - 3^2$
$= 5 - 3 \times 4 + 24 \div 8 - 3^2$ (**B**racket is removed and $3 + 5$ replaced with 8)
$= 5 - 3 \times 4 + 24 \div 8 - 9$ (**O**rder means p**O**wer – in this case $3^2 = 3 \times 3 = 9$)
$= 5 - 3 \times 4 + 3 - 9$ (**D**ivision $24 \div 8 = 3$)
$= 5 - 12 + 3 - 9$ (**M**ultiplication $-3 \times 4 = -12$)
$= 8 - 12 - 9$ (**A**ddition $5 + 3 = 8$)
$= \mathbf{-13}$ (**S**ubtraction $8 - 12 - 9 = -13$)

In practice, **it does not matter if multiplication is performed before division or if subtraction is performed before addition**. What is important is that **the process of multiplication and division must be completed before addition and subtraction**.

1.5.2 Brackets and operators

The basic laws governing the **use of brackets and operators** are shown by the following examples:

(a) $2 + 3 = 3 + 2$, i.e. the order of numbers when adding does not matter;

(b) $2 \times 3 = 3 \times 2$, i.e. the order of numbers when multiplying does not matter;

(c) $2 + (3 + 4) = (2 + 3) + 4$, i.e. the use of brackets when adding does not affect the result;

(d) $2 \times (3 \times 4) = (2 \times 3) \times 4$, i.e. the use of brackets when multiplying does not affect the result;

(e) $2 \times (3 + 4) = 2(3 + 4) = 2 \times 3 + 2 \times 4$, i.e. a number placed outside of a bracket indicates that the whole contents of the bracket must be multiplied by that number;

(f) $(2+3)(4+5) = (5)(9) = 5 \times 9 = 45$, i.e. adjacent brackets indicate multiplication;

(g) $2[3 + (4 \times 5)] = 2[3 + 20] = 2 \times 23 = 46$, i.e. when an expression contains inner and outer brackets, **the inner brackets are removed first**.

Here are some further problems where BODMAS needs to be used.

Problem 11. Find the value of $6 + 4 \div (5 - 3)$

The order of precedence of operations is remembered by using the mnemonic BODMAS. Thus

$$\begin{aligned} 6 + 4 \div (5 - 3) &= 6 + 4 \div 2 && \textbf{(B}\text{rackets)} \\ &= 6 + 2 && \textbf{(D}\text{ivision)} \\ &= \mathbf{8} && \textbf{(A}\text{ddition)} \end{aligned}$$

Problem 12. Determine the value of $13 - 2 \times 3 + 14 \div (2 + 5)$

$$\begin{aligned} 13 - 2 \times 3 + 14 \div (2 + 5) &= 13 - 2 \times 3 + 14 \div 7 && \textbf{(B)} \\ &= 13 - 2 \times 3 + 2 && \textbf{(D)} \\ &= 13 - 6 + 2 && \textbf{(M)} \\ &= 15 - 6 && \textbf{(A)} \\ &= \mathbf{9} && \textbf{(S)} \end{aligned}$$

Problem 13. Evaluate $16 \div (2 + 6) + 18[3 + (4 \times 6) - 21]$

$$\begin{aligned} &16 \div (2+6) + 18[3 + (4 \times 6) - 21] \\ &= 16 \div (2+6) + 18[3 + 24 - 21] && \textbf{(B}\text{, inner bracket is determined first)} \\ &= 16 \div 8 + 18 \times 6 && \textbf{(B)} \\ &= 2 + 18 \times 6 && \textbf{(D)} \\ &= 2 + 108 && \textbf{(M)} \\ &= \mathbf{110} && \textbf{(A)} \end{aligned}$$

Note that a number outside of a bracket multiplies all that is inside the brackets. In this case,

$18[3 + 24 - 21] = 18[6]$ which means $18 \times 6 = 108$

Now try the following Practice Exercise

Practice Exercise 4 Further problems on order of precedence and brackets (answers on page 531)

Evaluate the following expressions:

1. $14 + 3 \times 15$
2. $17 - 12 \div 4$
3. $86 + 24 \div (14 - 2)$
4. $7(23 - 18) \div (12 - 5)$
5. $63 - 8(14 \div 2) + 26$
6. $\frac{40}{5} - 42 \div 6 + (3 \times 7)$
7. $\frac{(50 - 14)}{3} + 7(16 - 7) - 7$
8. $\frac{(7 - 3)(1 - 6)}{4(11 - 6) \div (3 - 8)}$

For fully worked solutions to each of the problems in Exercises 1 to 4 in this chapter, go to the website:
www.routledge.com/cw/bird